MODELING THE COMPLEX DYNAMICS OF LANDSCAPE DEVELOPMENT: APPLICATIONS FOR LAND MANAGEMENT

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ABSTRACT

Erosion on military lands limits Army training activities, and may cause environmental problems both at training sites and downstream. Thus, remedial soil and land conservation plans are vitally important to stabilize erosion and use the land more safely for Army activities while complying with environmental regulations and standards. The channel-hillslope integrated landscape development (CHILD) model is a dynamic, physically-based, spatially distributed numerical framework for modeling soil erosion and landscape evolution, originally developed at MIT. Potential uses of CHILD include understanding the sensitivity of earth surface to changes in climate, land cover conditions and anthropogenic disturbances. In this paper, we present recent advances in the CHILD model with example numerical simulations. These are the effects of vegetation-erosion coupling on the short and long-term response of erosion rates and landscape topography, and modeling gully development by plunge-pool erosion at the gully head and widening by bank failures. CHILD simulations using a dynamic plant growth component show vegetation loss as an important factor accelerating erosion rates in decadal time scales. Over the long term, vegetation may change the dominant erosion process on the landscape, create episodicity in the sediment yields, and alter the visual appearance of landforms. Landscapes modeled using the new gully erosion module in CHILD produce landforms that compare well with the observed gullies in the Army training areas in Fort Carson and Pinon Canyon Maneuver Sites in Colorado. These results show potential applicability of the model for short and long-term land management purposes in the army lands.

1. INTRODUCTION

Landscape morphology results from the interaction of a complex set of nonlinear processes involving basin hydrology, geomorphic transport and landscape ecology. Despite this inherent complexity in the evolution of landforms, signatures of the dominant erosion and sediment transport processes, and the information on the geologic history is often preserved in the topography itself.

Numerical landscape evolution modeling serves a number of important purposes. By providing a crucial link between measurable processes at the small scale, and their long-term geomorphic implications over the large-scale, modeling offers an excellent opportunity to decipher geologic history of landforms, and provide the guiding impetus behind the development of quantitative hypothesis and field studies. In addition, serving as a numerical laboratory, computer modeling offers the ultimate platform to investigate the dynamic feedbacks among different physical processes and their self-organizing behavior, which may sometimes result in a counter-intuitive behavior (Tucker et al., 2001).

Needless to say, understanding the nature of complex landscape processes has an important relevance on the practical land management applications in much shorter time scales. To this end, computer modeling is often used as quantitative decision-support tool, which allows evaluating the performance of different land management strategies on environmental quality issues such as reducing basin sediment yields and managing flood risks. Landscape models are also in the interest of the U.S. Army. Erosion on military lands limits Army training activities, and may cause environmental problems both at training sites and downstream. Thus, remedial soil and land conservation plans are vitally important to stabilize erosion and use the land more safely for Army activities while complying with environmental regulations and standards.

In this paper, we report recent developments in the CHILD model, with examples relevant to land management problems within the U.S. Army Maneuver sites in Fort Carson and Pinon Canyon in Colorado where the landscape is undergoing rapid gully erosion. In what follows we first give a brief overview of the CHILD model. Then, a physically based model for gully development is introduced and its sensitivity to external forcing mechanisms and environmental factors is presented. Finally we present numerical simulation experiments conducted using the CHILD model evaluating the effectiveness of landscape vegetation, as a remedial land management measure.

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2. THE CHILD MODEL

The channel and hillslope integrated landscape development (CHILD) model simulates the evolution of the topographic surface resulting from changes in the landscape elevation under a set of driving erosion and sedimentation processes. The model represents the landscape topography by a set of nodes connected to form a triangulated irregular network mesh (TIN) (Figure 1).

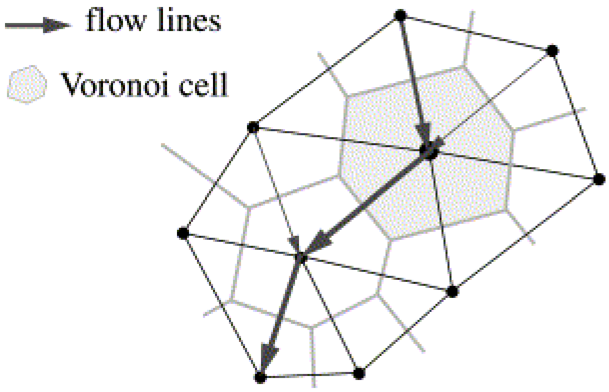


Figure 1. Elements of the irregular computational mesh, where, nodes (solid circles) are connected by triangle edges (black lines) to produce Voronoi polygons (after Tucker et al., 2001).

In CHILD, climate forcing is driven by a sequence of discrete rainfall pulses with durations, intensities and inter-arrival times. Runoff production is via infiltration excess and saturation excess runoff generation mechanisms. Surface runoff and sediment is routed through the triangulated mesh towards one or multiple of its adjacent neighbor nodes. Geomorphic processes incorporated in CHILD include; erosion of sediment or bedrock by running water, transport of sediment by slope dependent soil creep, landsliding, gully formation and overbank sedimentation on floodplains. The effects of these various geomorphic processes on the landscape can be mathematically represented by a generic partial differential equation;

$$\frac{\partial z}{\partial t} = U([x,y,z]) - F(Q,S,[Q_s,V]) - H(z) - G(Q,G_D,C) + D_{OB} + D_E \quad (1)$$

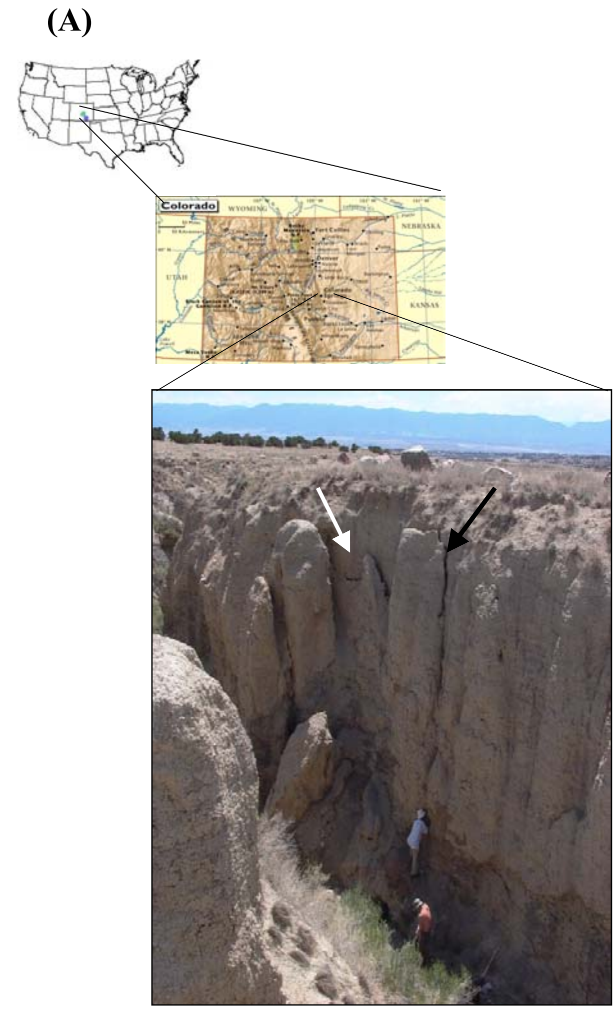
where, z represents surface elevation, t time, U tectonic uplift, F and H the erosion or deposition resulting from fluvial processes and hillslope processes, respectively, G gully erosion, and the last two terms represent overbank deposition during floods and eolian deposition. The fluvial processes depend on the runoff discharge, Q, surface slope in the direction of the flow, S, and optionally on the sediment flux, Q_s, and vegetation cover, V. Gully erosion is a function of runoff discharge Q, height of gully banks, G_D, and soil cohesion, C.

3. INVESTIGATING THE DYNAMICS OF GULLY FORMATION USING CHILD

A gully is an incised, steep-sided channel with an eroding headcut and slumping sidewalls (Schumm et al, 1984). Gully development is a highly dynamic process driven by feedbacks between terrain, vegetation, and hydrology. Gully erosion is often attributed to changes in external and internal factors in the basin. External factors determine the magnitude of flow shear stress or stream power acting on the soil surface. These include, tectonic processes (uplift, base level fall), climate forcing and natural and anthropogenic watershed disturbances. Watershed disturbances usually increase runoff production and/or reduce erosion resistance of the soil surface. Common watershed disturbances include road building that result in the alteration of topographic flow paths and removal of the protective surface vegetation cover due, for example, to grazing, forest clearing and wildfires (Prosser et al, 1995; Istanbulluoglu et al., 2004).

Gully erosion on military lands limit army training activities, poses threat to life and equipment and may also cause sedimentation problems downstream. Remedial soil and land conservation plans are needed to stabilize gully erosion and improve the safety in army activities. In Fort Carson and Pinon Canyon Maneuver Site (PCMS), Colorado, various remedial works have been undertaken to stabilize gully erosion including banksloping, revegetating and bank protection. However, such remediation efforts are hampered by the lack of understanding of the dynamics of gully erosion.

Among the gully development processes, widening due to mass-wasting of gully sidewalls appear to be the most widespread. A good example to this phenomenon is the development of large arroyo systems in Southwestern US, particularly in New Mexico, Arizona and Utah, that have incised since the 19th century. Figure 2A shows a deep gully eroded in the study area located in a military training site in Fort Carson, CO. Gullies in this area primarily erode by mass failures of their sidewalls. Such failures trigger as udercutting by fluvial erosion increases the bank height, producing stress concentration on the toe and at the top of the gully bank. Stress concentration in conjunction with the drying and wetting of the soils produce tension cracks at the back of the scarp. In Figure 2A, the black arrow marks a tension crack that has developed at the top of the gully bank and continued downward into the soil profile. As banks over-heighten, tension cracks also deepen to a depth where they intersect a potential failure plane that passes through the slope base with an inclination angle, producing a soil wedge (see Figure 2B). Slab failures occur when gravitational forces exceed the shear strength produced by the soil. The failed



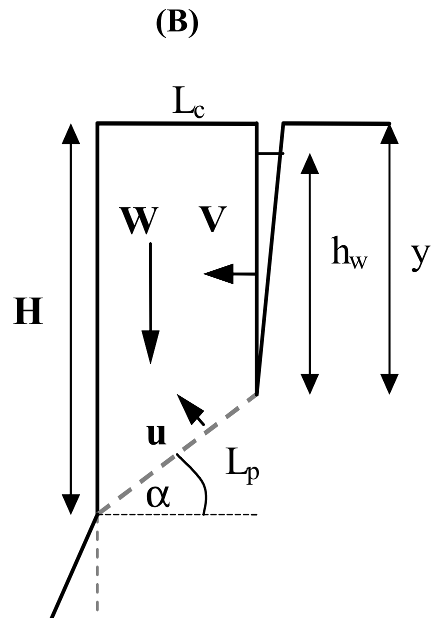


Figure 2. (A) Location of the field area in Fort Carson, Colorado mapped in the upper part of the figure. Below, a deep gully subject to gravitational failures, surveyed for model tests.

(B) Schematic illustration of the assumed geometry for an over-heightened gully bank (i.e., Figure 2A) and the forces that are acting on the potential failure block.

material deposits on the toe of the bank. The bank remains stable unless the deposit is removed by runoff erosion and rainsplash. Erosion of the toe material gradually exposes the vertical bank and makes the bank susceptible to another cycle of instability (Thorne, 1999).

Current level of understanding of gully erosion, especially with respect to the quantitative aspects, is still lacking. Gully erosion is a highly dynamic process that involves major changes in the landscape morphology that often creates a self-enhancing feedback between basin hydrologic and geomorphic processes. The dynamic nature of gully erosion makes it difficult to estimate soil loss due to gullying using the existing erosion models, which traditionally assume a fixed topography.

2. 2. A physically based model for gully formation

In order to extent the capabilities of the CHILD model as a numerical tool for basic geomorphic research and land management, we have implemented a physically based model for the stability analysis of gully head and side-walls within the CHILD model. The theory is based on the force-balance equation of an assumed planar failure geometry of a vertical gully wall, with a potential failure plane dipping to the incised gully bed (Figure 2B).

Based on the Coulomb equation, the factor of safety (FS), the ratio of resisting to driving forces acting on the failure plane shown in Figure 2B is expressed as:

$$FS = \frac{CL_p + (W\cos\alpha - U - V\sin\alpha)\tan\phi}{W\sin\alpha + V\cos\alpha} \quad (2)$$

where C is soil cohesion acting along the failure plane (Pa), L_p is the length of the failure plane (m), W is the weight of the block, U is hydrostatic uplift force due to positive pore pressure along the failure plane caused by water seeping from the crack, V is hydrostatic pressure of the water in the crack and ϕ is soil friction angle. Both U and V are related to the water depth in the tension cracks (see Figure 2A and 2B).

In the model, we relate the water level in the tension cracks during runoff producing storms to steady-state basin runoff production, assuming runoff is generated uniformly in the basin, and overland flow rate is proportional to a runoff and drainage area. When tension cracks are either dry or completely filled with runoff water, instability occurs when headcut height exceeds a critical threshold (higher for the dry case). For the case when cracks are partially filled, our theory predicts an inverse relationship between headcut height and drainage area. The stability model can be cast in the form of an instantaneous sediment transport law per unit width of scarp that has a discontinuity as;

$$q_s = \begin{cases} L_c(H + y_c)/2 & FS < 1 \\ 0 & FS \geq 1 \end{cases}, \qquad (3)$$

multiplying the unit failure volume with failure width gives the sediment flux to channels due to slab failures.

In the application of the slab failure model in CHILD, in a voronoi cell, a scarp face is assumed to be located on the side facing towards the flow direction. For convenience in the model description, a voronoi cell that hosts a scarp face is termed as a “scarp cell”. A scarp cell is identified when; (1) its local slope, approximated by forward difference of elevation, is greater than the slope of its downstream cell, and (2) cell height, estimated as the elevation difference between the cell and its downstream neighbor is greater than a maximum critical height for slabs with saturated cracks.

2.3. Modeling gully development using CHILD

Soils, climate and erosion thresholds are among the most important factors that control landscape evolution. Here we present results from our numerical model experiments conducted to explore the effects of soil cohesion, erosion thresholds (i.e., vegetation cover), and frequency and magnitude statistics of storms on gully development and resulting landscape morphology. In the simulations reported below, the rate of local erosion is modeled as a power function of discharge and slope in excess of an erosion threshold.

The initial condition used in the model is a 300 m wide, 400 m long and 10 m high rectangular block represented by a TIN mesh of 5 m node spacing, with one width-side set as an open boundary. Surface slope is set to 1% toward the open boundary. A natural analog to this initial condition would be rapid scour of a gently sloped valley floor by, for example, a mountain torrent.

The storm intensity, storm duration and interstorm duration are simulated stochastically using the Poisson pulse rainfall model. Figure 3 presents numerical model simulations using different soil cohesion values as 5 kPa, 10 kPa and 20 kPa respectively. With a mean rainfall intensity of 15 mm/h, mean storm duration of 1.7 h, and an interstorm duration of 18.5 days. Each figure contains the same amount of soil mass.

In the case of high soil cohesion (Figure 3c), deeper, but narrower gullies with vertical sidewalls develop compared to the simulations with lower soil cohesion. Landscape erodes primarily by wash erosion when soil cohesion is low, leading to the development of much wider valleys (Figure 3a). Another important conclusion from these runs is that increase in soil cohesion slows down the erosion process. Comparing the two-end member simulations, we find that a four-fold increase in cohesion resulted in approximately two orders of magnitude slower erosion rates.

Our numerical experiments are close analogues to the laboratory experiments of Howard (1988), designed to study landform development driven by groundwater sapping. Howard (1998) used both cohesionless and slightly cohesive sand, where he observed the growth of micro-gully walls due to backwasting, primarily driven by undermining due to seepage erosion and episodic mass-wasting processes. Introducing cohesion resulted in the development of narrower, but deeper gullies, and more dendritic gully networks. Erosion rates were also observed to be significantly lower in the case of cohesive material compared to the simulations conducted with cohesionless sand.

It has been suggested that differences in storm characteristics, such as storm intensity, frequency and seasonality rather than the mean annual precipitation is the primary factor in arroyo development in the Southwest US (Molnar, 2001). This hypothesis is tested by altering the mean rainfall intensity and duration, but keeping the total depth of random precipitation delivered by each storm invariant between simulations. This allows us to explore the sensitivity of gully development on storm intensity-duration characteristics with little change in the mean annual precipitation.

Figure 4 shows the effects of the differences in storm intensity and duration on landscape development. From left to right, there is a four-fold decrease in the storm rate and a four-fold increase in storm duration. Topography in the left figure shows channel widening due to gullying and talus slope formation, the one on the right (long-duration, low-intensity storms), however does not exhibit intense gully erosion, instead shows channels eroded by wash processes. Note that in both cases slab failures occur in the initial time slices of the simulations. Rapid removal of deposits in the high intensity, short duration simulation enhances the trigger of gravitational bank failures that results in widespread gully erosion in the domain. However when rainfall intensity is lowered, gully sidewalls stabilize due to a decrease in the frequency of erosive flood events. This dramatically reduces the fluvial response time scale, while in the meantime slope dependent diffusive processes (i.e., soil creep) shape the topography producing rounded hilltops.

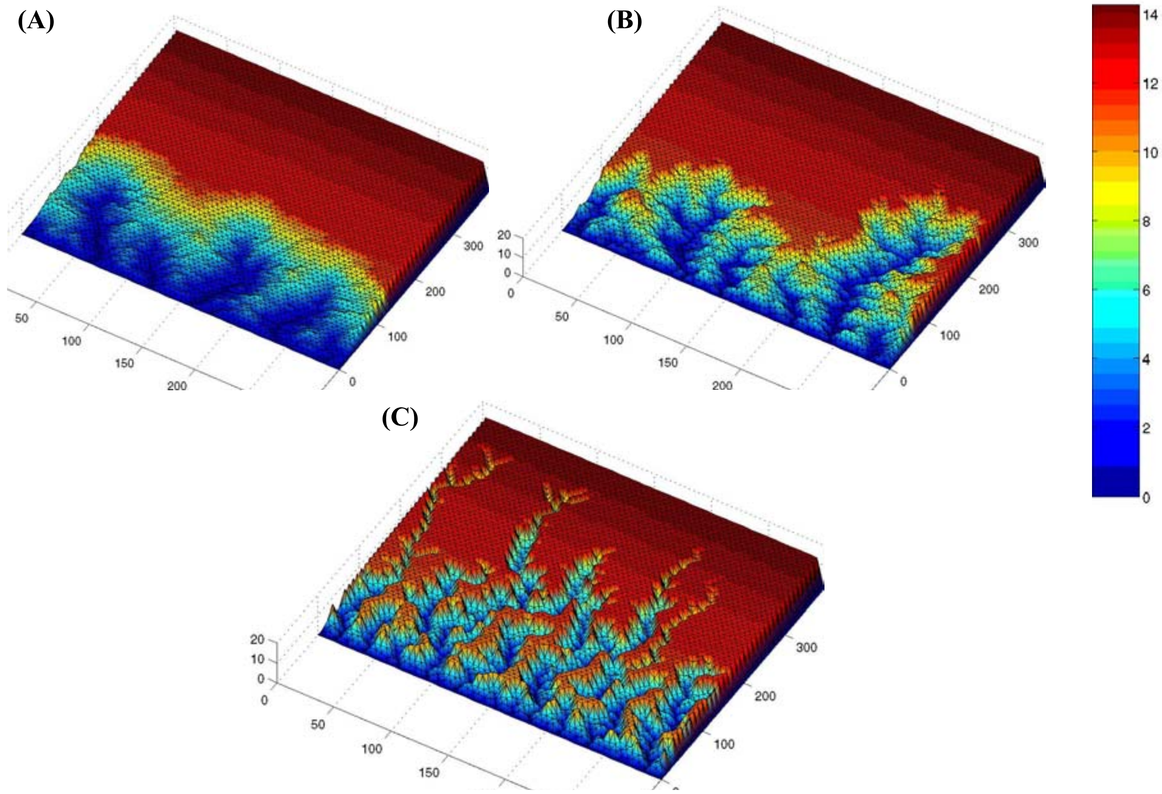


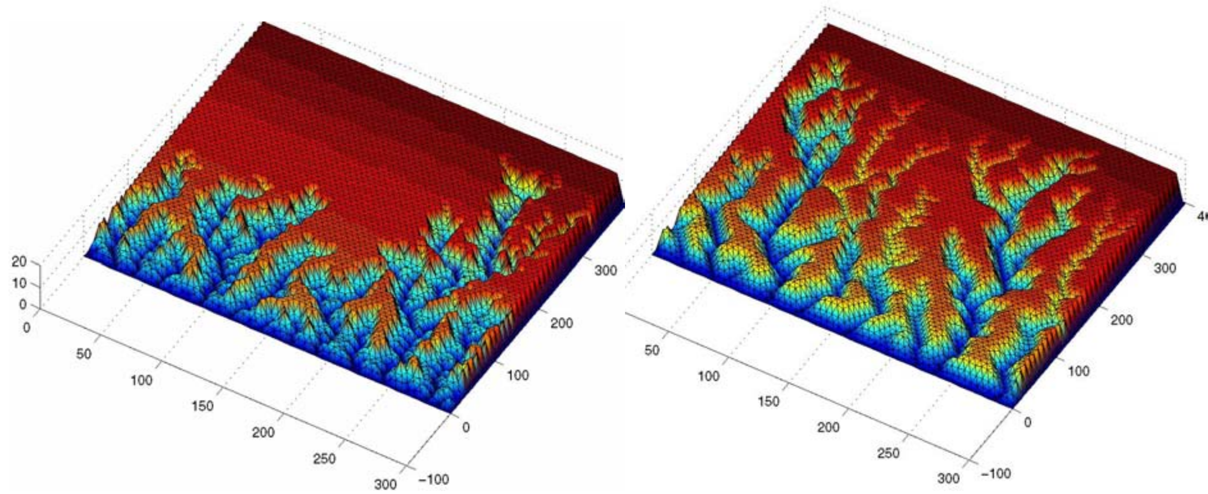
Figure 3. Topography after 20% of the initial mass is eroded. Soil cohesion values used, and the time slice in the model run correspond to (A) C=5kPa, time: 60 years (B) C=10 kPa, time: 100 years and (C) C=20 kPa time: 1100 years. Length is in meters.

Effects of the erosion threshold on gully development are also explored. Here we report results from three CHILD simulations, where sensitivity of the model is investigated with a two-fold increase, a two-fold decrease in the erosion threshold, and with no threshold. Here an increase in the erosion threshold corresponds to denser, static surface vegetation or the opposite is true for a decrease in the erosion threshold. No erosion threshold implies a bare soil surface of very fine, easily detachable material.

Landscape response to changes in erosion threshold is very similar in style, but in the opposite direction, to modeled changes in rainfall rate. Lowering the threshold enhances widespread gullying. Absence of any threshold completely erases the topography very rapidly (not pictured). When the erosion threshold is doubled, gully banks stabilize because of a reduction in the transport of the failure material from the toe slopes, resulting with a landscape eroding by soil creep and sheet wash (Figure 5).

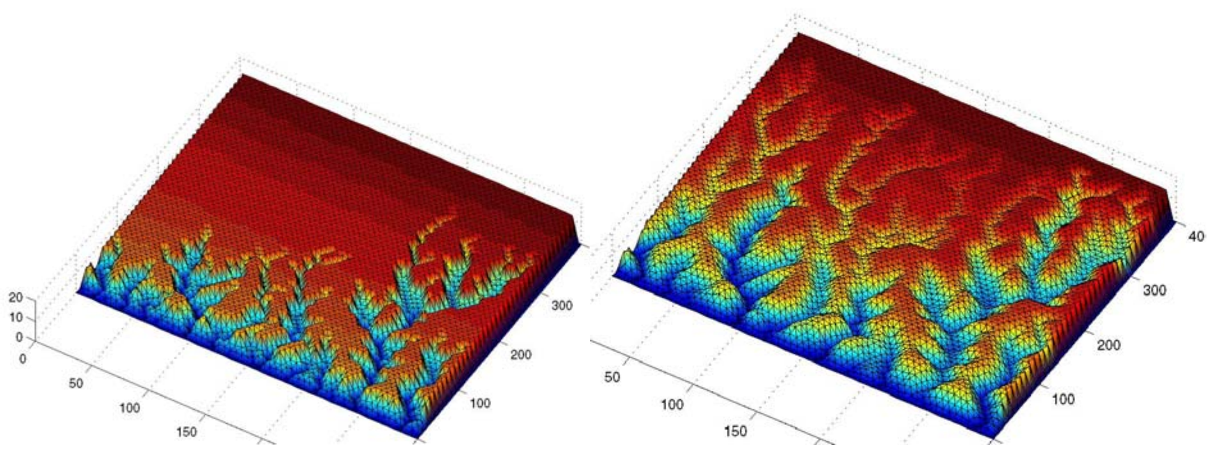
One interesting outcome of the model is that, similar to the experiments with different storm characteristics, initial landscape response to an abrupt change in the base-level is by gravitational failures despite a higher erosion threshold. When the erosion threshold is high (or storm intensity is low), gully walls are buttressed by slump debris and quickly stabilize, whereupon soil creep and soil wash processes eliminate the steep gully banks (Figure 5). After the cessation of bank failures, the landscape predominantly erode by soil creep and surface runoff, producing more rounded hillslopes.

As our simulations experiments suggest, gullies may form as a response to changes in the external controls in the basin, such as a rapid fall in the base level, as simulated in the numerical experiments. However, the tempo and form of erosion, and the resulting landscape morphology is shown to be controlled by soils, storm intensity and duration characteristics and erosion thresholds. These predicted differences in the landscape response to the above mentioned factors may allow for developing strategies for managing gully erosion. In the next section, we have provided an example in which effectiveness of revegetating, as a remediation measure, in controlling gully erosion is forecasted using the CHILD model.



(A) **(B)**

Figure 4. Topography after 20% of the initial mass is eroded. For climate driven by: **(A)** High-intensity short-duration storms **(B)** low-intensity, long duration storms. In both case soil cohesion is 20 kPa and mean annual precipitation is constant. Dimensions are in meters.



(A) **(B)**

Figure 5. Landscapes showing the effects of a two-fold increase in runoff erosion threshold: **(A)** Low threshold **(B)** High threshold. In both cases soil cohesion is 20 kPa and mean annual precipitation is constant. Dimensions are in meters. Note the development of rounded hilltops in (B), as the role of soil creep becomes more pronounced in shaping the landscape under a higher runoff erosion threshold.

4. MODELING VEGETATION – EROSION DYNAMICS

4. 1. Implications of vegetation on land form evolution

Vegetation plays an important role in erosion and landscape evolution. Vegetation influences fluvial erosion directly, by increasing surface resistance to wash erosion, reducing the effectiveness of runoff shear stress in soil detachment and providing extra cohesion to soil by its roots. Vegetation significantly alters the frequency and magnitude of watershed hydrology via its influence on infiltration and runoff generation processes and evaporation losses.

In CHILD, plants, if present, are assumed to form a uniform ground cover within a voronoi cell (Figure 1) increasing the thresholds required for runoff erosion and landsliding. Vegetation grows as a function of both available cover and unoccupied space by plants according to the well-known logistic equation (Levins, 1969), and is killed by geomorphic disturbances (runoff erosion and landsliding), and wildfires. The growth rate increases as

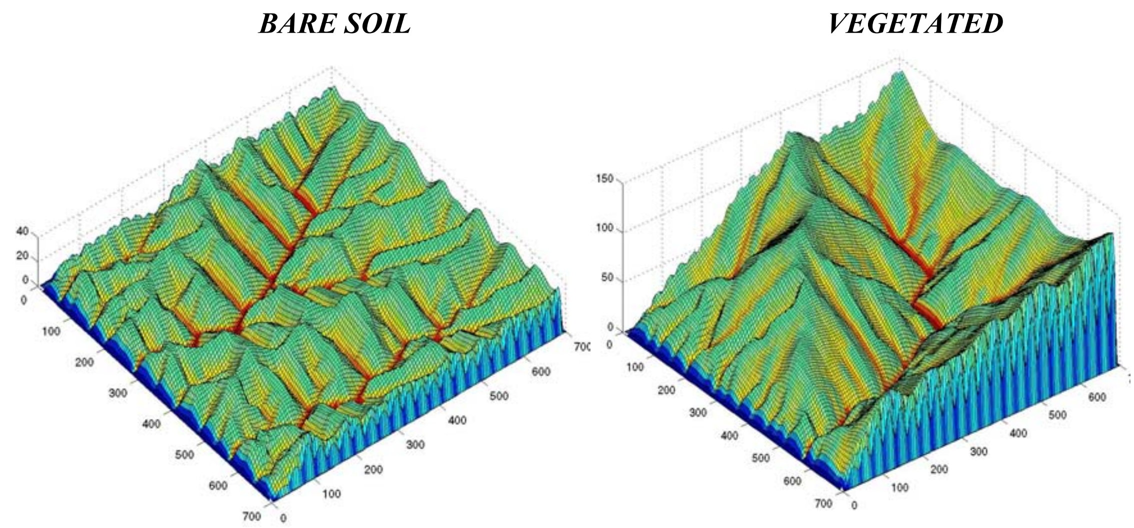


Figure 6. Simulated landscapes for bare soil and vegetated land surface conditions.

vegetation becomes denser, due to the availability of seeds and dispersal sources. Vegetation growth approaches zero either when the cover is very dense, or sparse. In the former case space limits the plant growth whereas in the latter, sources for reproduction limits growth.

Dynamics arising from the interactions between vegetation and erosion results in fundamental differences in the spatial dominance of various erosion processes and resulting landscape morphology (Collins et al., 2004; Istanbulluoglu and Bras, 2004). To illustrate, we present two end-member numerical simulations using the CHILD model (Figure 6). In the first simulation, no vegetation is allowed to grow on the landscape. In the second, vegetation grows with a negligible mortality rate and is killed by floods and mass failures. Both simulations are driven by stochastic storms and tectonic forcing, representative of the Oregon Coast. In the simulations, vegetation cover causes a shift in the dominant erosion process. Hillslopes devoid of vegetation erode primarily by soil wash forming a highly dissected topography. On the other hand, vegetation seals the soil surface, inhibiting wash erosion. Continuing uplift increase the slopes to a point, at which slopes destabilize and erode by landsliding. Development of steep hillslopes increase the basin relief significantly altering the visual appearance of the landscape.

4. 2. Effects of vegetation in gully erosion control

In this section, we present a set of numerical experiments that explore the effectiveness of grass in gully erosion control. In the initial condition, gully formation has commenced in the lower parts of the domain and is propagating upstream as a response to a sudden base level fall, in the absence of landscape vegetation. Two different types of grass are "planted" in two separate simulations (Prosser et al., 1995). Both provide the same level of surface resistance and erosion threshold, but one is more susceptible to erosion than the other. The model is run for several decades. As pictured in Figure 7, compared to the no vegetation case, both grass types provide significant erosion control. While resistant grass is slightly more effective in reducing the propagation of gullies than the less resistant, limited erosion continues in both cases.

Based on flume experiments conducted in a natural hillslope, Prosser and coworkers (Prosser et al., 1995) suggested that thresholds imparted by natural grass cover could be so high that channels could only initiate under unrealistic discharges. Our simulation experiments presented in Figure 7 underscore the value of numerical modeling of the dynamic interactions among different landscape processes. Despite the large erosion threshold generated by vegetation, gully erosion continues at a reduced pace because of other external factors, such as increased relief via base level fall and randomness in the storm forcing.

5. CONCLUSIONS

Landscapes are dynamic features, shaped under the influence of a number of highly nonlinear physical and biological processes. Because of this complexity, investigating the landscape behavior requires a system approach, in which different landscape processes interact. The CHILD landscape evolution model advances the state of the art in numerical modeling of landscape development by integrating a wide variety of processes. This paper has presented examples illustrating the use of CHILD as both a research and a decision making tool.

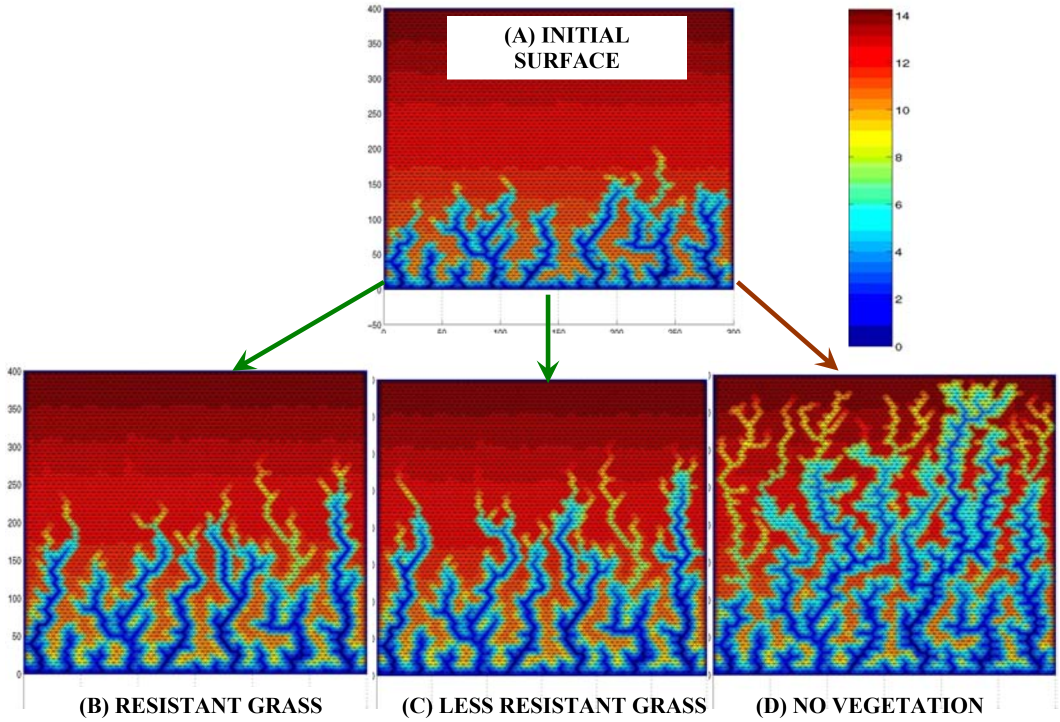


Figure 7. Effects of grass plantation on the upstream propagation of gully erosion. Initial surface is devoid of vegetation, underlain by cohesive soils.

Our findings, relevant to landuse planning, underscore the importance of dynamic landscape modeling in forecasting the effectiveness of soil conservation plans, while suggesting a longer term perspective in planning soil and land resources conservation projects.

6. ACKNOWLEDGEMENTS

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